

Numerical simulation of photocatalytic oxidation of gaseous pollutants using low Reynolds number turbulence models

Shijian Peng, Bensheng Zhao, Haiyan Zhang, Jia Wang, Baoqing Deng*

Department of Environmental Science and Engineering, University of Shanghai for Science and Technology, Shanghai 200093, P R China

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CORRESPONDING AUTHOR:

bqdeng@usst.edu.cn
Tel: 00862155271991
Fax: 008655271991

ABSTRACT

Introduction: Photocatalytic oxidation of gaseous pollutants in differential reactors is simulated using computational fluid dynamics.

Materials and methods: The momentum equation and pollutant transport are solved by using ANSYS Fluent. The SIMPLE algorithm is used to treat the pressure-velocity coupling. The laminar flow and low Reynolds $k-\epsilon$ models are used to describe turbulence.

Results: Velocity field distribution and degradation efficiency of different models at various flow rates were obtained and compared with the experimental data. The simulation results of degradation efficiency under different models are basically consistent.

Conclusion: Although low Reynolds $k-\epsilon$ models have better simulation results for high inlet flow rates, in terms of computation complexity, laminar flow is recommended for simulation.

Introduction

Photocatalytic oxidation technology has attracted significant attention in the field of pollutant control [1, 2]. Under the illumination of UV light, photocatalytic reaction is activated through the absorption of photons with the energy greater than the band-gap energy, producing an electron-hole pair [3–5]. A series of reduction and oxidation reactions are followed transforming pollutants into carbon dioxide and water.

Hydrodynamics plays an important role in photocatalytic reaction, which directly influences the transport of pollutants within the reactor. An effective methodology to understand the hydrodynamics is computational fluid dynamics (CFD),

in which the continuity and momentum equations are usually solved by using finite volume method. Annular reactors with simple geometry have been successfully simulated with laminar flow [6–8] and turbulent flow [9, 10]. For reactors with multiple lamps or complex catalyst surface, CFD simulations depict good ability adapting to complex internal configurations under the laminar flow [11–13]. When the catalyst exists as particles, multiphase flow was simulated in reactors in combination with suitable turbulence models [14–18]. In general, good agreement was obtained between the experimental data and the simulations for these laboratory-scale reactors.

When simulating the photocatalytic reaction in

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laboratory scale reactors, the kinetic rate equation must be known, which is usually described by the Langmuir-Hinshelwood equation. The rate constants in the Langmuir-Hinshelwood equation are measured in differential reactors [6, 19–22]. Differential reactors are very small and contains a plane surface coated with the catalyst. By using the assumption of perfect mixing, the rate constants can be obtained from the experiment in differential reactors [19, 22]. On the other hand, laminar flow [23] or turbulent flow [20] were also used to simulate differential reactors and evaluate rate constants. However, no assessment on the effect of laminar and turbulent flows was conducted.

The simulation of differential reactors is crucial to determine the rate constants. In this work, laminar flow and several low-Re $k - \varepsilon$ turbulence models are used to simulate a differential reactor. The aim is to determine the suitable model for the evaluation of rate constants of photocatalytic reactions in differential reactors.

Materials and methods

Flow equations

For the photocatalytic reaction in differential reactors, the air is usually used as carrier gas. Air flow in the reactor obeys the conservation law of mass and momentum. The aim of differential reactors is to measure the reaction rate. Therefore, air flow is usually maintained at steady state. Under the steady condition, the continuity equation and Navies-Stokes equation can be written as follows:

$$\frac{\partial(\rho u_j)}{\partial x_j} = 0 \quad (1)$$

$$\frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial u_i}{\partial x_j} - \overline{\rho u'_i u'_j} \right) \quad (2)$$

where ρ is the fluid density, and x_j and u_j are the coordinate component and velocity component in the j direction, p is the pressure, μ is the fluid ki-

nematic viscosity, is the turbulent stress.

Low-Re $k - \varepsilon$ models

For laminar flow, the turbulent stress in Eq. 2 is zero. When turbulence must be considered, the turbulent stress can be described as

$$-\overline{\rho u'_i u'_j} = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \rho k \delta_{ij} \quad (3)$$

where k is the turbulent kinetic energy, μ_t is the turbulent eddy viscosity.

For laboratory scale reactors, the near-wall turbulence is of great importance because of low Reynolds number. Therefore, low-Re $k - \varepsilon$ models are used to describe the turbulence effect. The transport equations of turbulent kinetic energy and its dissipation rate are written as follows

$$\frac{\partial(\rho u_j k)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k - \rho \varepsilon - \rho D \quad (4)$$

$$\frac{\partial(\rho u_j \varepsilon)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{\varepsilon 1} f_1 \frac{\varepsilon}{k} G_k - C_{\varepsilon 2} f_2 \rho \frac{\varepsilon^2}{k} + E \quad (5)$$

with

$$\mu_t = C_\mu \rho f_\mu \frac{k^2}{\varepsilon} \quad (6)$$

where C_μ , σ_k and σ_ε are empirical constants, ε is the turbulent dissipation rate, f_μ , f_1 and f_2 are damping functions. D and E are near-wall correction terms for k and ε equations.

Several low-Re $k - \varepsilon$ models have been developed, which are designated as ABID for Abid [24], LB for Lam and Bremhost [25], LS for Launder and Sharma [26], YS for Yang and Shih [27], AKN for Abe and Kondoh [28] and CHC for Hsieh and Chang [29]. Damping functions for these low-Re $k - \varepsilon$ turbulence models are listed in Table 1.

Degradation equation of pollutants

Ethylene is selected as the model pollutant. After

entering the differential reactor, ethylene moves by the convection and diffusion. The governing equation of ethylene in the reactor is expressed by:

Table 1. Damping functions in turbulence models

Model	f_μ	f_1	f_2
ABID	$\tanh(0.008Re_y)(1 + \frac{4}{Re_t^{3/4}})$	1.0	$[1 - \frac{2}{9}\exp(-\frac{Re_t^2}{36})][1 - \exp(-\frac{Re_y}{12})]$
LB	$[1 - \exp(-0.0165Re_y^2)]^2(1 + \frac{20.5}{Re_t})$	1.0	$1 - \exp(-Re_t^2)$
LS	$\exp(-\frac{3.4}{(1 + Re_t/50)^2})$	1.0	$1 - 0.3\exp(-Re_t^2)$
YS	$[1 - \exp(-1.5 \times 10^{-4}Re_y - 5.0 \times 10^{-7}Re_y^3 - 1.0 \times 10^{-10}Re_y^5)]^{1/2}$	$\frac{\sqrt{Re_t}}{1 + \sqrt{Re_t}}$	$\frac{\sqrt{Re_t}}{1 + \sqrt{Re_t}}$
AKN	$[1 + \frac{5}{Re_t^{0.75}}\exp(-(\frac{Re_t}{200})^2)][1 - \exp(-\frac{y^*}{14})]^2$	1.0	$[1 - 0.3\exp(-(\frac{Re_t}{6.5})^2)][1 - \exp(-\frac{y^*}{3.1})]^2$
CHC	$[1 - \exp(-0.0215Re_y)]^2(1 + \frac{31.66}{Re_t^{5/4}})$	1.0	$[1 - 0.01\exp(-Re_t^2)][1 - \exp(-0.0631Re_y)]$

Note: $Re_t = \frac{k^2}{\nu\varepsilon}$, $Re_y = \frac{yk^{1/2}}{\nu}$, $y^* = \frac{y\varepsilon^{1/4}}{\nu^{3/4}}$

$$\frac{\partial}{\partial x_j}(u_j C) = \frac{\partial}{\partial x_j} \left[\left(D + \frac{\nu_t}{\sigma_t} \right) \frac{\partial C}{\partial x_j} \right] \quad (7)$$

where C is the ethylene concentration, and D is the molecular diffusion coefficient of ethylene in the gas phase, σ_t is the turbulent Schmidt number, ν_t is the kinematic turbulent viscosity.

In differential reactors, catalysts are coated onto the wall of reactor. Thus, the photocatalytic oxidation of ethylene only occurs at the wall. For walls with catalyst, the boundary condition can be written as follows

$$-\left(D + \frac{\nu_t}{\sigma_t} \right) \frac{\partial C}{\partial n} = r \quad (8)$$

where n is the outward normal direction at the boundary. The photocatalytic rate is usually described using Langmuir-Hinshelwood equation as follows:

$$r = \frac{k'KC}{1 + KC} \quad (9)$$

where k is the rate constant, K is the adsorption constant. Eqs. 8, 9 comprise the boundary condition of Eq. 7 as follows

$$-\left(D + \frac{\nu_t}{\sigma_t} \right) \frac{\partial C}{\partial n} = \frac{k'KC}{1 + KC} \quad (10)$$

Eq. 10 should be realized in an iterated manner since the diffusion flux of ethylene to the wall changes during iteration.

For walls without catalyst, there exists

$$-\left(D + \frac{\nu_t}{\sigma_t} \right) \frac{\partial C}{\partial n} = 0 \quad (11)$$

which means that no ethylene can penetrate into the wall.

Numerical simulation

A rectangular reactor with the dimensions of 54.5 mm × 54.5 mm × 47.5 mm is studied, as shown in Fig. 1. A glass plate with a square hollow is positioned on the bottom of reactor. Catalysts are coated onto the bottom of hollow. Polluted gas enters into the reactor and is oxidized at the surface of catalyst. Some researchers measured the degradation efficiencies under different inlet flow rates and inlet concentrations [20]. ANSYS ICEM is used to discretize the computational

domain, as shown in Fig. 2. Near the surface of catalyst, the grid is refined. The total number of mesh is 565,392.

The governing equations are solved by using ANSYS Fluent. SIMPLE algorithm is adopted to deal with pressure-velocity coupling. Convection terms are discretized by using the second-order upwind scheme. The concentration equation is described by using user defined scalar (UDS). The solution was considered convergent when the scaled residuals for all equations are less than 10^{-6} .

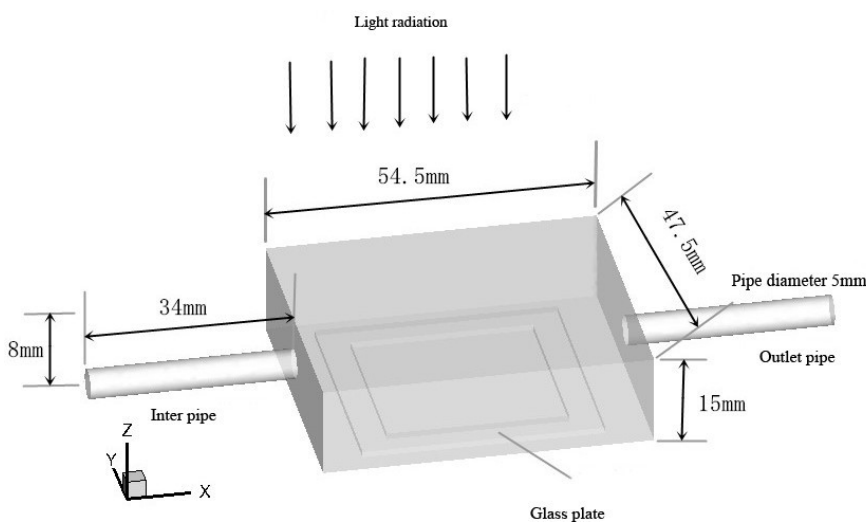


Fig. 1. Schematic of differential reactor

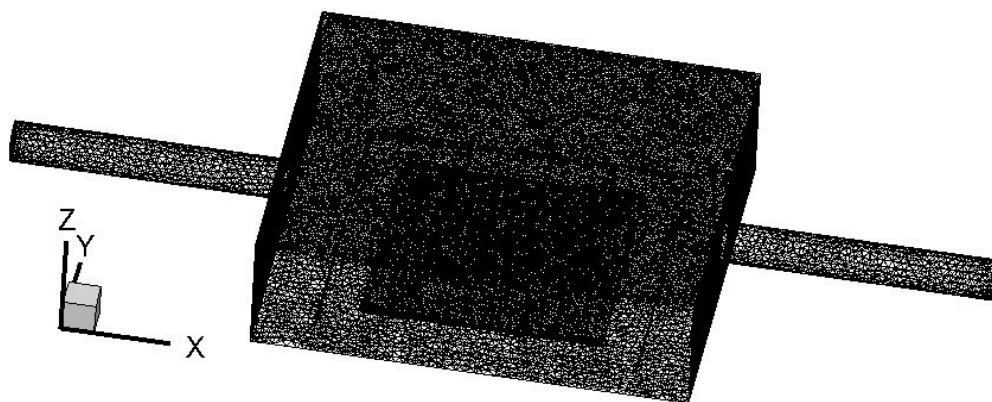


Fig. 2. Schematic of grid

At the inlet, volumetric flow rate and the concentration of pollutant are given. The selected inlet flow rates are 50 mL/min, 100 mL/min, 150 mL/min and 250 mL /min and the corresponding Reynolds numbers are 14.5, 29.0, 43.6 and 72.6, respectively. The turbulence intensity is set as 10 %. Inlet concentrations of ethylene are set as 2.11×10^{-6} , 4.17×10^{-6} , 8.33×10^{-6} , 1.23×10^{-6} and 1.63×10^{-6} mol/L, respectively. At the outlet, pressure is specified. No-slip condition is specified at all walls.

Results and discussion

Velocity distribution

Fig. 3 shows the distribution of gas velocity for laminar flow. After gas enters the reactor, vortices are formed beside two side walls of reactor, which is typical of a sudden expansion flow. In the central part of the reactor, velocities depict a uniform feature. Near the reactor outlet, the flow depicts a feature of sudden contraction. With the increase in flow rate, the vortices increase in

magnitude. The vortex is restricted in a small region for 50 mL min⁻¹ whereas distributes full of the entire reactor cross-section for 250 mL min⁻¹. The velocity distributes more uniformly when the flow rate is low. Along the direction from inlet to outlet, the velocity is largest, meaning that the flow tends to be directed to the outlet. This is because the fluid is affected by the viscous stress at the wall while the shear stress at the center line is very small.

Fig. 4 shows the velocity at the center line from inlet to outlet. As shown in Fig. 4(a), When gas enters the reactor at a flow rate of 50 mL/min, the velocity is 0.0428 m/s with an error 0.9%. The maximum velocity at the center of the pipeline can reach 0.0795 m/s, which is about twice the averaged velocity. The distribution of velocity in the reactor is very non-uniform. Fig. 4(b) shows that the velocity is largest at the center of pipe, and gradually decreases toward the pipe wall, presenting a symmetrical distribution. The velocity under other flow rates are similar to each other.

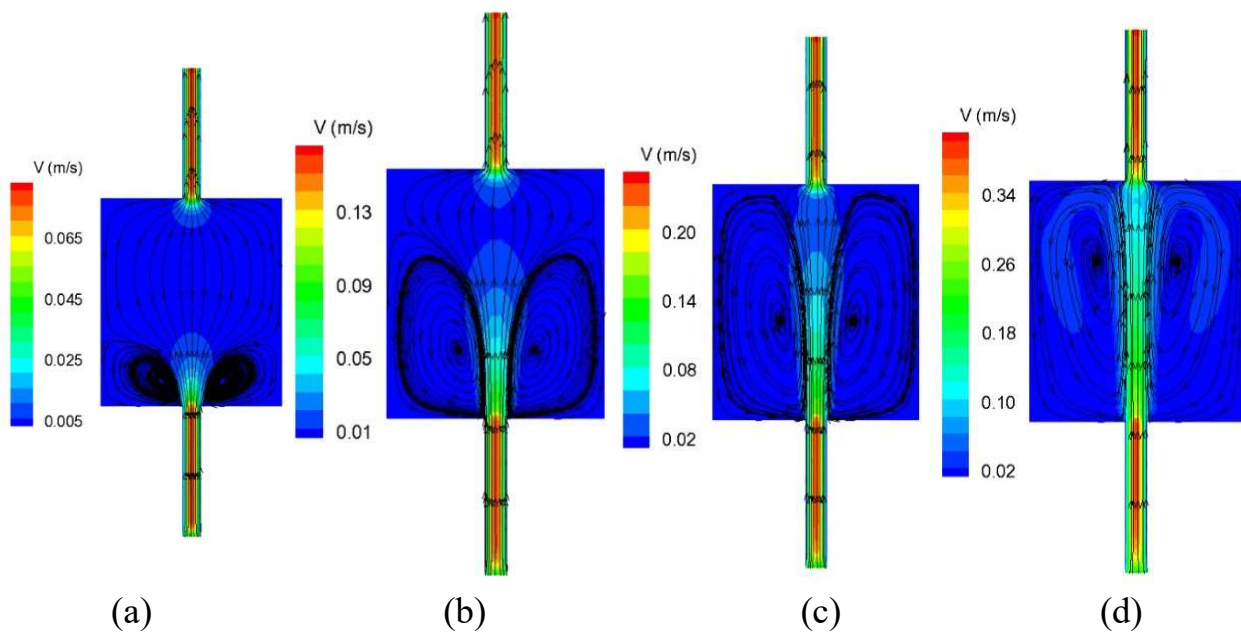


Fig. 3. Velocity distribution for laminar model at the plane of $z = -0.5$ mm: (a) 50 mL/min, (b) 100 mL/min, (c) 150 mL/min, (d) 250 mL/min

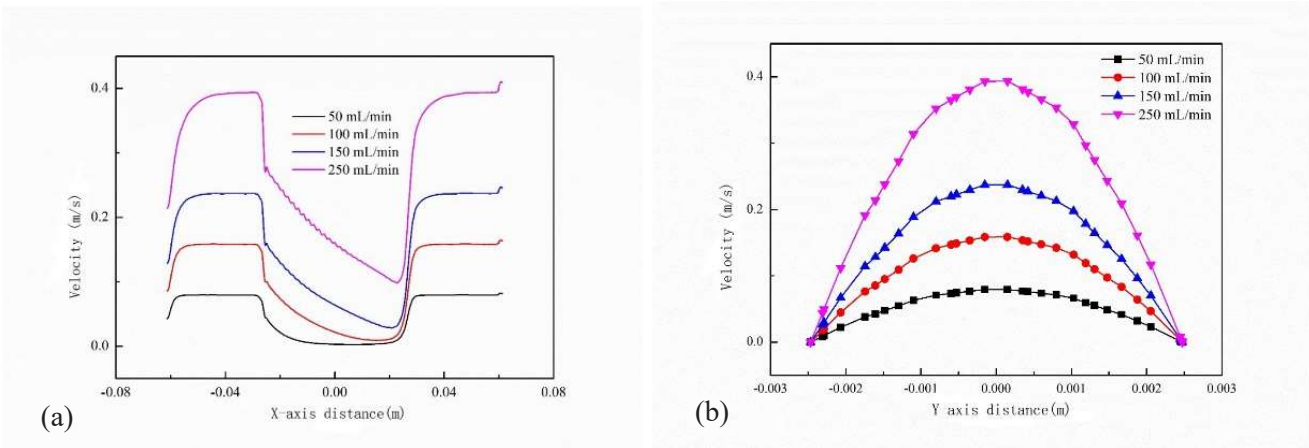


Fig. 4. Velocity profile of laminar model:
 (a) ($y=0, z=-0.5$ mm), (b) ($x=50$ mm, $z= -0.5$ mm)

Fig. 5 shows the distribution of gas velocity by using the AKN low-Re $k-\epsilon$ model, respectively. For 50 mL/min, vortices occupy a half of the cross-section in the reactor, which is greater than that for laminar flow. Correspondingly, the region with uniform velocity is small. The flow patterns for 250 mL/min are similar for laminar flow and two turbulence models. The velocity distributions for other low-Re $k-\epsilon$ models are similar to those of the AKN models and will not be described in detail here.

Fig. 6 shows the velocity profile at the center line from inlet to outlet. When flow rates are small, the velocity obtained by six low-Re $k-\epsilon$ model are basically consistent. As the flow rate increases, the fluid velocity also gradually increases and the velocity distributions of the six low Reynolds number models at the outlet pipe are different. When the flow rate reaches 250 mL/min, LS, YS and AKN models obtain nearly the same velocity distribution inside the reactor.

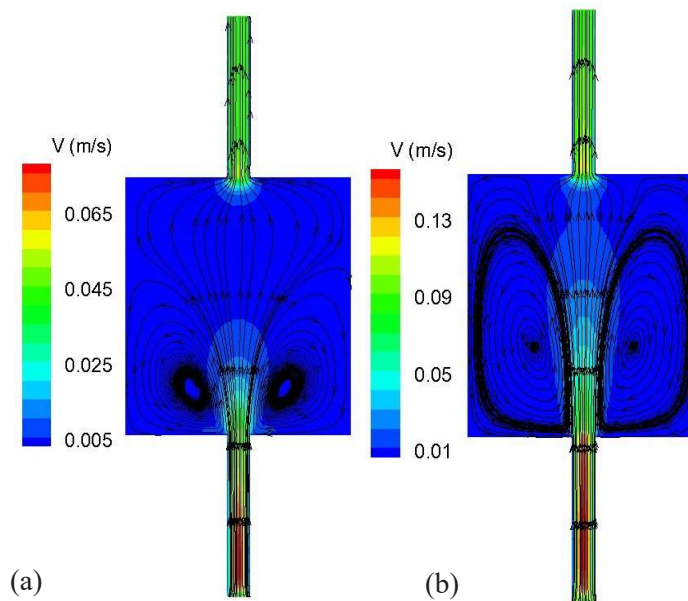


Fig. 5. Velocity distribution for the AKN model at the plane of $z = -0.5$ mm:
 (a) 50 mL/min, (b) 250 mL/min

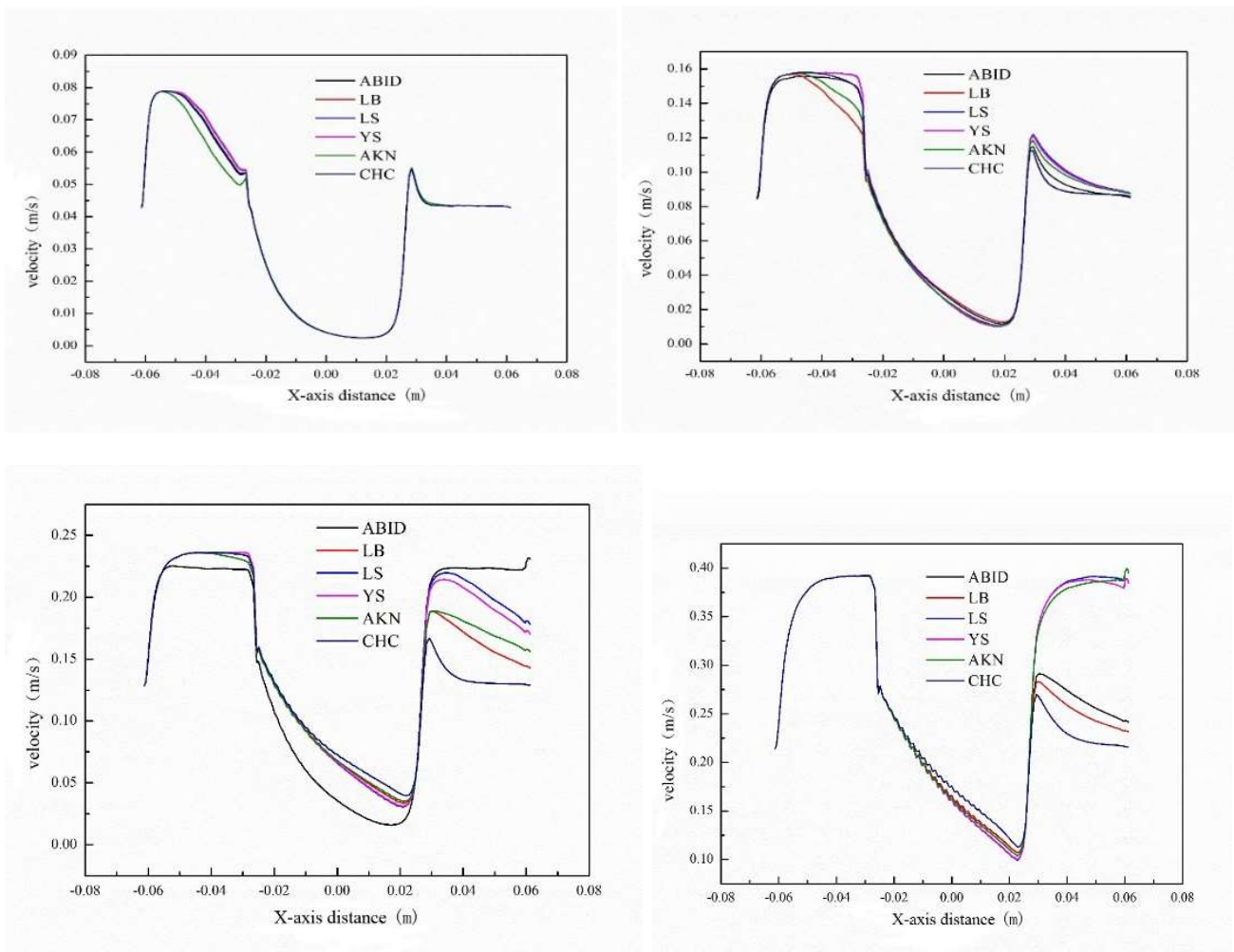


Fig. 6. Velocity profile at the position of ($y = 0, z = -0.5$ mm):
 (a) 50 mL/min, (b) 100 mL/min, (c) 150 mL/min, (d) 250 mL/min

Degradation efficiency

Degradation efficiency is an important index to evaluate numerical simulations, which reflects the integrated effects of flow and photocatalytic reaction in differential reactors. Table 2 and Table 3 show the degradation efficiencies under different inlet concentrations for inlet flow rates of 50 mL/min and 250 mL/min, respectively. The experimental values from some researchers are also included in these two tables. For the inlet flow rates of 50 [20], all numerical simulations obtained the nearly identical result [20]. The maximum relative error appears at a low inlet con-

centration of 50 ppm. With the increase in inlet concentrations, the relative error decreases. The same tendency is observed for the inlet flow rates of 250 mL/min. In general, degradation efficiencies obtained by numerical simulation are consistent with the experimental values. All low-Re turbulence models behave better for small inlet concentrations whereas the laminar model predicts the better result for big inlet concentrations. In terms of degradation efficiency, all these models can be used to simulate pollutant degradation in the present photocatalytic reactor.

Table 2. Degradation efficiency under different models at 50 mL/min (%)

ethylene concentration (ppm)	50	100	200	300	400
experimental value	30.40	21.92	12.07	9.85	8.03
Laminar	23.22	17.51	11.28	8.37	6.75
ABID	23.69	17.63	11.32	8.37	6.61
LB	23.70	17.63	11.32	8.37	6.61
LS	23.70	17.63	11.32	8.37	6.61
YS	23.70	17.63	11.32	8.37	6.61
AKN	23.71	17.63	11.32	8.37	6.61
CHC	23.70	17.63	11.32	8.37	6.61

Table 3. Degradation efficiency under different models at 250 mL min⁻¹(%)

ethylene concentration (ppm)	50	100	200	300	400
experimental value	7.10	4.68	2.44	1.85	1.50
Laminar	5.02	3.62	2.29	1.71	1.35
ABID	5.19	3.69	2.30	1.69	1.33
LB	5.19	3.69	2.30	1.69	1.33
LS	5.19	3.69	2.30	1.69	1.33
YS	5.19	3.69	2.30	1.69	1.33
AKN	5.19	3.69	2.30	1.69	1.33
CHC	5.21	3.70	2.30	1.69	1.33

The rate constants of ethylene are fitted directly by using Eq. 9 [20]. The estimated values are $k = 1.3 \times 10^{-2} \text{ mol/ms}^2$ and $K=480 \text{ m}^3/\text{mol}$. Fig. 7 shows the degradation efficiency for laminar flow using these rate constants. In general, the computed degradation efficiency accords well with the experimental values. However, there exists some discrepancy for small inlet flow rates. This error can be reduced by optimizing the reaction rate constant. The optimized values of k and K are $1.52 \times 10^{-2} \text{ mol/ms}^2$, and $630.0 \text{ m}^3/\text{mol}$. As shown in Fig. 8, the laminar model reproduces the experiment for both 50 mL/min and 250 mL/min using the optimized parameters.

Conclusion

Numerical simulations of flow and reaction in differential reactors are conducted based on laminar flow and six low Re turbulence models. All simulations obtain similar flow fields. The flow in reactors is governed by vortex near the inlet. The vortex regions differ much for low inlet flow rate whereas are similar for high inlet flow rates. The degradation of ethylene is considered. The degradation efficiencies predicted by all simulations are in accordance with the experimental values. The optimization of rate constants may improve the simulation. Considering the computation complexity, laminar flow can be recommended to simulate the photocatalytic reaction in differential reactors.

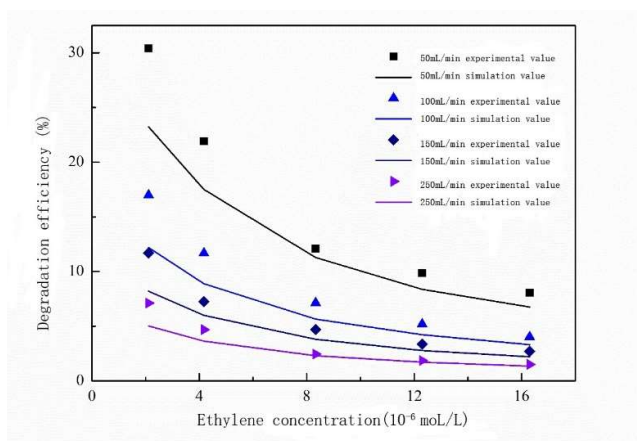


Fig. 7. Degradation efficiency of ethylene for laminar model

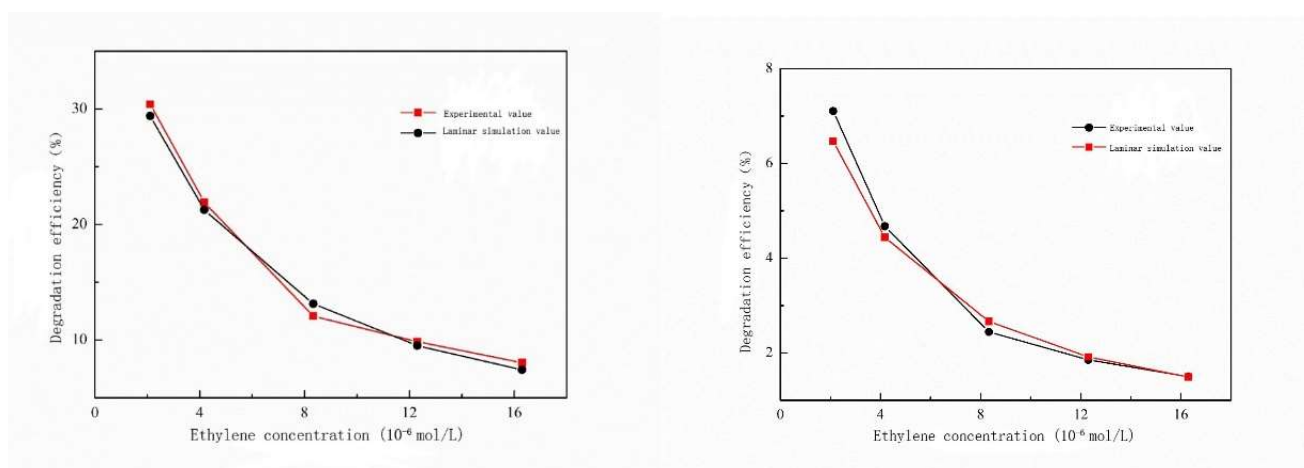


Fig. 8. Degradation efficiency of ethylene for laminar model with optimized parameters:

(a) 50 mL/min, (b) 250 mL/min

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Competing interests

The authors declare that there are no competing interests.

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Ethical considerations

“Ethical issues (Including plagiarism, Informed Consent, misconduct, data fabrication and/or falsification, double publication and/or submission, redundancy, etc.) have been completely observed by the authors.”

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